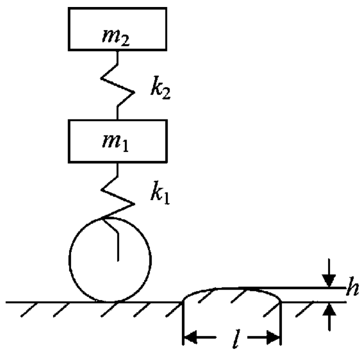

Fig. 1 System model.

$(\mu_l, \sigma_l) = (3, 0.15)$ m, respectively, and $v = 20$ m/s. From experience, the specified range of γ should be chosen between 10 and 15% of the first natural frequency.

The differential equation of the vehicle system is

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = k_1h \sin pt$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0$$

The probability of the quasi-failure state of the whole system is

$$P_f = 0.0$$

Thus, resonant vibration cannot occur in the system.

It is obvious that vehicle runs on the rough road avoiding velocities of 7.309 m/s or 63.694 m/s. Here the probability of the quasi-failure state of whole system is

$$P_f = 0.9774$$

Thus, resonant vibration can occur in the system.

Conclusions

Frequency analysis is important for reliability problems of dynamically uncertain structures. This Note presents a frequency reliability analysis method. According to the criterion of the frequency relation, the structural system with resonant failure is defined as a series system. One of the reliability problems of random structural systems is preferably solved using frequency analysis and reliability theory. Numerical results are presented.

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Topology Optimization for Maximum Natural Frequency Using Simulated Annealing and Morphological Representation

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I. Introduction

FOR topology optimization of continuum structures, the most common strategy is based on discretizing the allowable space into finite elements (FEs) and considering the material density or amount of material within each element as design variables. When the relevant optimization problem is solved, the optimum geometry thus emerges from the resulting optimum distribution/arrangement of material within the design space. The homogenization[1] and material density[2] methods are the popular methods based on such a strategy, and many static and dynamic topology/shape optimization problems have been solved using them.[3–5] However, because this strategy uses continuous design variables, the resulting elements can be of intermediate densities (gray areas) spanning the range from completely empty to completely filled with material. Hence, the final geometry has to be interpreted from these results, usually by imposing some threshold density value, which may be arbitrary and thus lead to uncertainties. Furthermore, with no controls or constraints over the arrangement of material, the optimum result may contain checkerboard patterns and floating elements disconnected from the main structural body.

To overcome these drawbacks, the recently developed morphological representation scheme[6] is used to define structural geometry in this work, optimizing structures to achieve maximum natural frequency with a constraint on the volume of material. Because changes in topology are discrete changes, the problem is treated as a discrete optimization using the simulated annealing (SA) algorithm inasmuch as the SA is readily adaptable to various classes of problems and has the ability to escape from local optima with the possibility of reaching the global optimum. The SA also does not enumerate

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too large a number of states during the optimization process and so is suitable for treating large system problems in a reasonable amount of time. It has been successfully applied to many problems.[7,8]

II. Methodology

A. Morphological Representation Scheme

In the morphological scheme,[6] the design problem is characterized by a set of input/output regions in the design domain, which is discretized into finite elements. Input regions typically are regions of the structure where prescribed boundary conditions, for example, displacement or loading conditions, are applied. Output regions are typically regions where the response of the structure is required or where some function performed by the structure is realized. These regions (predefined by the designer) must necessarily exist because any structure must have parts that interact with their surroundings by way of support conditions, loading and/or some other functional interactions. A representative illustration with three input/output regions in a plane rectangular domain is shown in Fig. 1a. The design domain is denoted by a design space within which the structure must lie and cannot exceed, and it is discretized into a mesh of quadrilateral FEs (Fig. 1b) with the required loading/boundary conditions defined on it.

For a valid structural design, all input/output regions must be connected to one another either directly or indirectly. Thus, the scheme is based on specifying a few paths (each joining one region to another) to form the structural topology. A parametric curve (such as the Bezier curve, which is used in this work) is needed to form each path connecting any two input/output regions (Fig. 1b). Each Bezier curve is defined by its start and end input/output points plus a number of control points in between. The start points and endpoints are the center of elements of the respective input/output regions. The control points are also at the center of the respective elements in which they are located. The set of elements through which each Bezier curve passes form the skeleton elements connecting the two regions (Fig. 1c). Some of the surrounding elements are then added onto the skeleton to fill up the structure to its final form. These elements represent the flesh elements surrounding the skeleton. Each Bezier curve is divided into a number of segments[6] according to the number of control points, and each segment has a corresponding thickness value to define the number of layers of flesh elements for that segment. This is done by considering each skeleton element and adding an all-round layer of flesh elements to it, with the layer thickness depending on the corresponding thickness value. The union of all skeleton, flesh, and input/output region elements together constitutes the structure, whereas all other elements remain as void (empty space).

The position of the input/output regions and control points is denoted by the column and row numbers of the element containing it, whereas the thickness values are denoted by an integer. Thus, the position of the control points and the thickness values together make up the design variables in the optimization problem. When the morphological scheme is used, the topology and shape of a structure is determined by the arrangement of the skeleton and flesh elements and is an outcome of the interaction among a few parametric curves and their respective thickness values. The scheme will not generate any invalid designs such as those with checkerboard pattern or disconnected floating elements. Hence, the topology/shape optimization problem is solved as a true discrete problem without the need for much further interpretation of any resulting design.

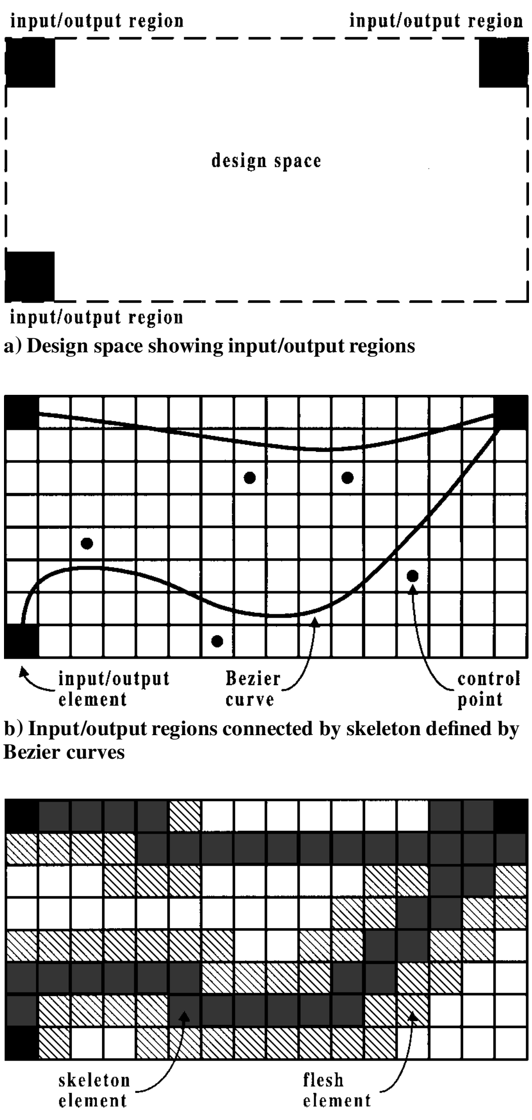


Fig. 1 Definition of structural geometry by morphological scheme.

B. Problem Statement and the Simulated Annealing Algorithm

The objective function of the topology optimization problem is the first natural frequency $f(\boldsymbol{x})$ of the structure, with one constraint being that the area $A(\boldsymbol{x})$ occupied by the structure is not greater than 40% of the design space and where $\boldsymbol{x}$ denotes the vector of design variables given by

$$\boldsymbol{x} = \left[x_1^1, y_1^1, x_2^1, y_2^1, x_3^1, y_3^1, \ldots, x_1^2, y_1^2, x_2^2, y_2^2, x_3^2, y_3^2, \ldots, t_1^1, t_2^1, t_3^1, \ldots, t_1^2, t_2^2, t_3^2, \ldots\right]^T \tag{1}$$

in which (x_i^j, y_i^j) is the column and row number of the ith control point of the jth Bezier curve, and t_k^l is the thickness value for kth section of the lth curve.

For the SA algorithm implemented in this work, new design states are created by random variations of these design variables within the neighborhood of any current design state. If the variation leads to an improved objective function value, the new design state can be accepted and become the current state. If the variation leads to an inferior objective value, the new state may still be accepted according to an acceptance probability P_0 expressed as a function of a decreasing control parameter T (temperature) as follows:

$$P_0 = \exp[(f^{\text{new}} - f^{\text{current}})/T] \tag{2}$$

where f^{new} and f^{current} are the objective functions of new and current design states, respectively, and T decreases according to a special cooling schedule developed by Johnson et al.[9]

In this work, a special constraint handling technique also has been developed based on the same annealing concepts applied to acceptance probabilities for inferior objective value design states. Here, an acceptance probability P_c for constraints is introduced[10] as follows:

$$P_c = \exp\left[\frac{40\% - R_{\text{area}}(\boldsymbol{x})}{\lambda T^2}\right] \tag{3}$$

where $R_{\text{area}}(\boldsymbol{x})$ is the percentage of the design space occupied by the current structure and λ is a temperature coefficient (for adjusting the rate of decrease of P_c as T decreases). The role of P_c is quite similar to that of P_0 in that, in the early stages when T is high and P_c is close to one, designs that violate the constraint can also be accepted. As T decreases, P_c decreases rapidly toward zero by which time no infeasible designs can be accepted. This technique facilitates exploration of the search space, even for complex constraints that give rise to scattered feasible regions, enhancing the advantage of the SA method in escaping from local optima.

III. Results and Discussion

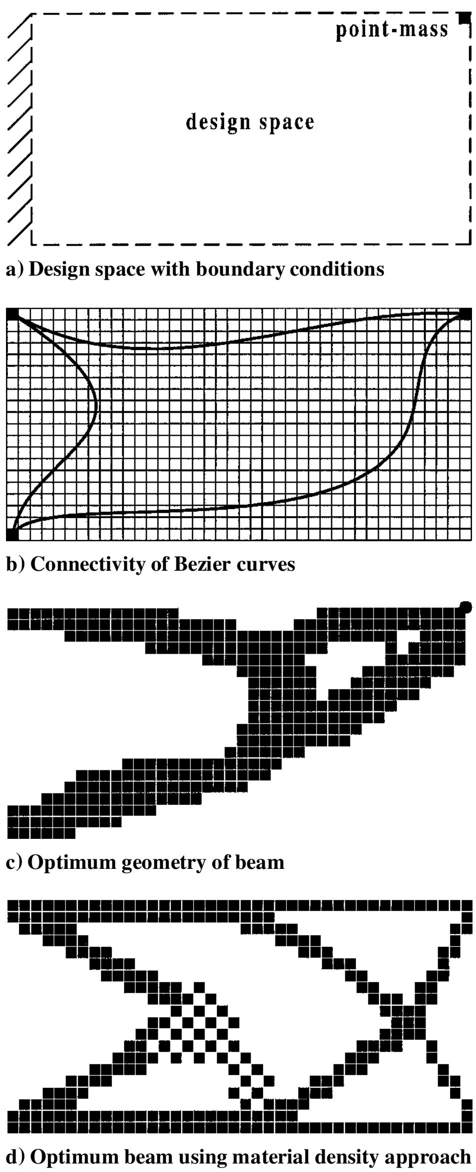
Two two-dimensional plane cantilever beam dynamic topology optimization examples have been solved, each with a 0.508×0.254 m design space, a uniform thickness of 0.0254 m, and material parameters of $E = 2.1 \times 10^{11}$ N/m^2, $\nu = 0.3$, and $\rho = 7800$ kg/m^3. In both examples, the design space is discretized into a 40×20 mesh of eight-node quadrilateral elements, with the left edge of the cantilever fully fixed and the right edge free. The ABAQUS FE software is used to evaluate the dynamic responses of the structure. A single point mass is attached at the top-right corner of the beam in the first example, and there are two point masses in the second example, one at the top-right and the other at the bottom-right corner. All point masses are 174.8 kg each.

A. Dynamic Topology Optimization for Cantilever Beam with One Point Mass

The design space and boundary conditions are shown in Fig. 2a. Three input/output regions are used to characterize this problem in the design domain. Three Bezier curves (each with five control points) are used to define the structural geometry, and a representative illustration of their connectivity is as shown in Fig. 2b. When the SA procedure is used, topology optimization is performed with a total of 48 design variables, and the optimum beam geometry obtained is shown in Fig. 2c with a first natural frequency of 46.9 Hz and a material usage of 39.9% of the design space. The same problem has also been solved using the material density approach of Yang and Chuang[2] and Yang and Chahande[5] with the optimum result shown in Fig. 2d. This result achieved a higher frequency of 51.5 Hz, but this may be because four-node elements were used, and these elements are stiffer than the eight-node elements used in the current method.

B. Dynamic Topology Optimization for Cantilever Beam with Two Point Mass

The design space and boundary conditions are shown in Fig. 3a. There are four input/output regions in the design domain. A representative illustration of the six Bezier curves (each with five control points) used to form the structural geometry is shown in Fig. 3b. The total number of design variables is 96, and the optimum beam obtained (Fig. 3c) has a frequency of 36.4 Hz with material usage at 40%. The same problem has again been solved using the material density approach, with the optimum beam (Fig. 3d) attaining a frequency of 44.0 Hz. On a more qualitative note, the resulting shapes from the two different approaches are quite similar for this problem, although they are quite different for the one point mass



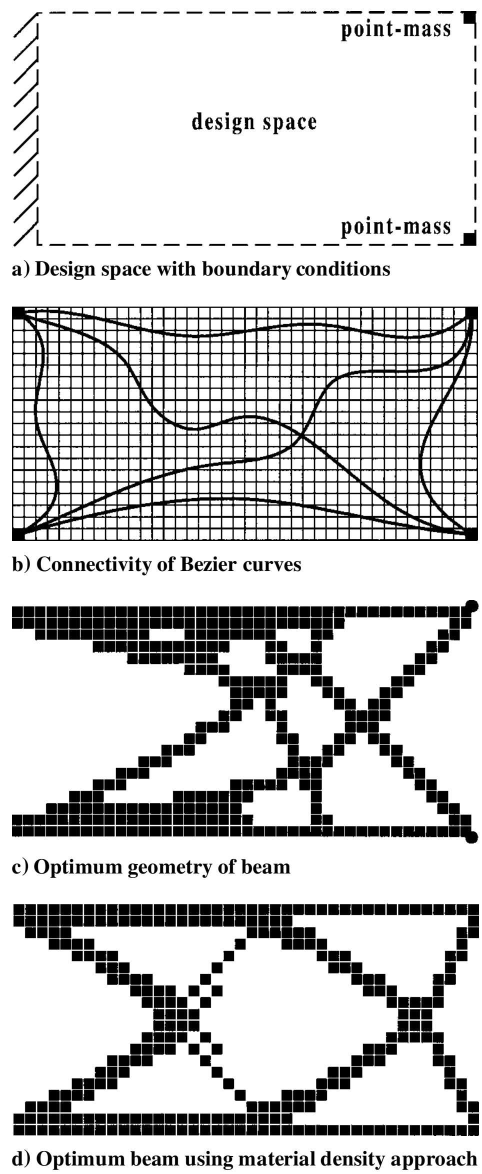
a) Design space with boundary conditions

b) Connectivity of Bezier curves

c) Optimum geometry of beam

d) Optimum beam using material density approach

Fig. 2 Cantilever beam problem with one point mass.



a) Design space with boundary conditions

b) Connectivity of Bezier curves

c) Optimum geometry of beam

d) Optimum beam using material density approach

Fig. 3 Cantilever beam problem with two point mass.

problem. In addition, the material density results exhibit checkerboard patterns (more severely in the one point mass problem), and this poses a problem when the structural design has to be realized and fabricated.

IV. Conclusions

Two-dimensional dynamic topology optimization problems have been solved to achieve maximum natural frequency of cantilever beam structures within a specified material usage. The morphological geometry representation scheme used here will not generate undesirable checkerboard patterns or floating/disconnected elements, ensuring that each design evaluated is a valid structure with little need for further interpretation of the resulting optimum geometry. The concept of acceptance probability for constraints was also introduced as a special strategy for the SA procedure to handle constrained optimization problems.

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